Abstract

In the paper, the properties of stochastic dynamical system consisting of random circle homeomorphisms are studied. The main assumption is that the system acts minimally (which implicates that its orbits are dense). Moreover, this system generates Feller operator as well as random walk on the circle. For the Feller operator, the existence of unique (ergodic) invariant measure has been shown. Furthermore, for Markov chain corresponding to the random walk on the circle, the central limit theorem and the law of iterated logarithm have been proven. These theorems are satisfied for arbitrary centered Lipschitz function and for every starting point of the Markov chain.

The first chapter introduces the main objects and theorems which are the subject of the research in further parts of the work.

In second chapter, the properties of stochastic dynamical systems on general polish or convex spaces are presented, including the properties of Markov-Feller operators and the existence of invariant measure.

The third chapter is devoted to the existence of unique invariant measure for the Feller operator generated by stochastic dynamical system consisting of random circle homeomorphisms and acting minimally. The e-property is used in the proof. Also, the examples of application of the main theorem as well as the sufficient conditions for the Feller operator to be asymptotically stable are presented.

The fourth chapter includes the proofs of the central limit theorem and the law of the iterated logarithm for Markov chain corresponding to the random walk generated by stochastic dynamical system consisting of random circle homeomorphisms and acting minimally. Both theorems are proved for arbitrary centered Lipschitz function and for every starting point of the Markov chain. In the proof of the central limit theorem, the results of Y. Derrienic and M. Lin were used, and in the proof of the law of the iterated logarithm, the results of O. Zhao and M. Woodroofe were applied. In this chapter, the example of application proven limit theorems is also presented.