

Abstract

Non-locality is one of the properties most exploited by quantum communication protocols to achieve a variety of purposes, such as key generation (QKD) for spatially separated agents or the generation of true random numbers that cannot be predicted by anyone in the universe and which can be exploited for cryptographic purposes.

Since Bell's seminal work, many studies have demonstrated the possibility of using quantum correlation to achieve better performance in the field of communication tasks than can be achieved with classical strategies.

Often the investigation of quantum strategies that offer the highest chance of success in a given task is very arduous and complex, sometimes even impossible with today's knowledge, making numerical approaches useful, and sometimes necessary.

In this thesis I show new contributions in the fields of entanglement witnessing, random number generation and tasks involving multiple agents acting on graphs, achieved with the aid of convex optimisation techniques. This study is carried out through the presentation of three of my articles.

In the first article, the quantum experiment proposed involves two trusted agents and an eavesdropping agent. This experiment addresses the problem of distinguishing entangled quantum states from separable quantum states through the use of the entanglement witnessing technique in the presence of imperfect communication and in a context in which the devices used are not fully trusted. In this work convex optimization techniques have been used to establish to what extent an eavesdropping agent can simulate the behaviour of entangled quantum states through the use of separable quantum states.

To deal with non-detection events, we applied techniques already known and studied in the context of Bell scenarios to the entanglement witnessing technique, and compared the results obtained with different approaches.

In the second article, the quantum experiment proposed involves two trusted agents and an eavesdropping agent. In this work we introduced a new algorithm and experimental setup that can be exploited to demonstrate non-local phenomena at very large and, under certain assumptions, arbitrarily large distances, in the presence of imperfect communication. This proposal involves the introduction and implementation of the so-called routed Bell experiment, in which, in one of its formulations, a source sends quantum states to an agent located near the source and to a switch which randomly sends its subsystem to another agent near the source or to one far from the source. The agents placed close to the source perform measurements on the received quantum

state trying to obtain correlations that maximally violate the CHSH inequality, while the agent placed further away performs measurements aiming to obtain outcomes which we have demonstrated to be random and non-reproducible through the use of local description in a non-conspiratorial context. In this project we used numerical convex optimization techniques to quantify the maximum predictive power of an adversary who has access to the source of the transmitted quantum states, thus quantifying the randomness obtainable as a function of the inefficiency of the devices used by the agent placed at large distance from the source.

In the task proposed in the third article, we show how sharing quantum states can increase the success probabilities of multiple agents performing communication task. In particular we studied some tasks implemented by mobile agents positioned on the vertices of a graph, focusing on different variants of the graph rendezvous problem and on different variants of a communication task we introduced, taking the graph domination problem as an example. We investigated a possible quantum advantage, i.e. an increase in the probability of victory, or more generally, in the score that agents can obtain when they share quantum states. In this study, convex optimisation methods were used to investigate the optimal quantum strategies that the agents can employ comparing them with what they can do adopting only classical strategies. We considered only tasks for which the success probability or the score is described by a linear function with respect to the distribution of the outputs obtained by the various agents.