Abstract

In this dissertation, we are concerned with finding an approximate solution to the linear partial differential equations with a highly oscillatory potential function and a strongly elliptic differential operator. We consider evolution equations involving first- or second-time derivatives, such as the heat equation or the wave equation. Such equations present significant challenges in numerical treatment and standard methods are mostly ineffective for them.

In the first part of the thesis, we analytically derive the Modulated Fourier expansion for a linear partial differential equation with a multifrequency highly oscillatory potential. The Modulated Fourier expansion (MFE) is an important technique in computational mathematics used, *inter alia*, to study the long–time behavior of Hamiltonian systems with highly oscillatory solutions. Moreover, MFE can be utilized in numerical-asymptotic approach as an ansatz for finding approximate solution to linear or nonlinear highly oscillatory differential equations. To derive the Modulated Fourier Expansion for the considered problem, we show that the solution of the equation is expressed as a convergent Neumann series in the appropriate Sobolev space. Then, by using integration by parts and the theory of semigroups, we expand asymptotically each of integrals from the Neumann series into a sum of known coefficients. By organizing terms appropriately we obtain formulas for the coefficients of the Modulated Fourier expansion. The proposed approach enables, firstly, to determine the coefficients for this expansion and secondly, to derive a general formula for the error associated with the approximation of the solution by MFE.

In the second part of the thesis we propose, using the results of the first part, a third-order numerical integrator based on the Neumann series and the Filon quadrature, designed mainly for highly oscillatory partial differential equations. The method can be applied to equations that exhibit small or moderate oscillations; however, counter-intuitively, large oscillations increase the accuracy of the scheme. The proposed approach enables the easy improvement of the method's accuracy and the straightforward estimation of its error.

The proposed computational methods are illustrated with many examples. For each equation in the numerical examples of this dissertation, we know the analytical solution. Thus, we do not need to compute reference solutions using a supercomputer. Using the proposed methods, we can accurately compare the numerical approximation with the function satisfying the equation.