Abstract

Non-classical features of quantum theory fuel a quantum over classical advantage by improving task performance or resource efficiency. The operational scenario underlying a task sets constraints within which some quantum resources resist efficient classical simulation and effectively lead to an advantage. Additionally, advantages in information processing tasks serve as tools to certify non-classical behaviour. This thesis investigates the advantages offered by non-classical resources in communication and non-local scenarios, and develops methods to detect non-classical quantum resources.

The first part introduces the *distributed clique labelling problem* for orthogonality graphs and studies the classical and quantum communication required for one-way zero-error *distributed computation* and *relation reconstruction* without shared resources. While distributed computation shows no quantum advantage for some graphs, relation reconstruction reveals an instance of *unbounded* quantum advantage. The classical communication cost without shared and private randomness grows as $\log_2 n$ bit with the order of the graph n and the classical cost without shared randomness is at least $\log_2 K(\mathcal{G})$ bit for graph $\mathcal{G}$ with disjointness number $K(\mathcal{G})$. $K(\mathcal{G})$ is lower bounded by $\max\{\omega(\mathcal{G}),\ \log_2 n + \frac{1}{2}\log_2\log_2 n + O(1)\}$. In contrast, the quantum cost is $\log_2 \omega(\mathcal{G})$ qubit whenever $\mathcal{G}$ admits a faithful orthogonal representation in dimension $\omega(\mathcal{G})$, where $\omega(\mathcal{G})$ is the clique number (size of the maximum clique in the graph). We identify families of graphs where the order n grows without bound, while the faithful orthogonal range and clique number remain constant. For these graphs, the quantum cost remains fixed while the classical cost grows without bound, establishing an unbounded quantum advantage in relation reconstruction. We also show that the necessary shared randomness for bounded classical communication scales as $\log_2(\lceil\log_2 \alpha\rceil + 1)$ bit with the number of maximum cliques α. For a specific family of graphs, this leads to an unbounded separation between entanglement assistance and shared randomness assistance required for a one-bit classical channel.

The second part presents *randomness-free* schemes for detecting *non-projective-simulable measurements* in scenarios with separated parties sharing systems with bounded local operational dimension. It presents detection schemes for three- and four-outcome qubit non-projective measurements in a bipartite scenario, some of which are robust against arbitrary depolarising noise. It also proposes schemes for detecting five-outcome qutrit non-projective measurements. In a bipartite case with identical devices, the work presents a scheme and provides evidence of its robustness against arbitrary depolarising noise. Relaxing the identical-device assumption, detection schemes for qutrit non-projective simulable measurements are discussed in both bipartite and tripartite scenarios. The work extends the notion of non-projective simulable measurements for General Probabilistic Theories and presents a randomness-free test to show that the square-bit model or box world theory is unphysical.

The third part studies correlation-assisted bounded classical communication tasks in a one-way prepare and measure scenario when the receiver is not given an input. It constructs a Bell inequality tailored to a correlation-assisted classical communication task with a linear payoff using the *wire-cutting* technique. The violation of this inequality implies an advantage in the corresponding task, and vice versa. It then introduces the concept of *wire-reading*, which leverages the readability of the classical messages, and uses it to present two families of tasks where shared randomness assistance gives a strictly suboptimal payoff. In the first, any correlation from a non-local facet of the no-signalling polytope achieves the optimal payoff, with explicit quantum advantage shown for some cases. In the second family, each task is tailored to a specific non-local facet, and correlations on that facet achieve the maximum payoff. Considering tasks tailored to non-local extremal correlations with dichotomic outputs, where the shared randomness-assisted payoff is at most 0.75, it shows any correlation on the isotropic line connecting the extremal point to white noise leads to an advantage as long as the noise fraction is below 0.5. It introduces a third family of tasks characterised by two parameters, demonstrating advantageous assistance from Hardy-type correlations with dichotomic inputs to a one-bit channel. While the payoff with shared randomness is bounded by zero from above, Hardy correlations yield a positive payoff. The maximum quantum payoff using Hardy-type correlations grows with a task parameter. Finally, it shows an instance when a two-qutrit entangled state achieves maximum quantum payoff. In contrast, two-qubit entangled states achieve payoffs higher than the local bound, but strictly lower than the quantum maximum, demonstrating a qutrit over qubit advantage. This provides an operational method to witness the local dimension of the shared entangled system.