Self-presentation

I. Name: Rafał Filipów

II. Scientific degrees

- Ph.D. in mathematics, Institute of Mathematics of the Polish Academy of Sciences, Warsaw 2004. Thesis title: *The difference property in de Bruijn sense for families of measurable functions.* Advisor: prof. dr hab. Ireneusz Recław.

- M.Sc. in Mathematics, Faculty of Mathematics and Physics of the University of Gdańsk, 2000. Thesis title: *The difference property for families of measurable functions.* Advisor: prof. dr hab. Ireneusz Recław.

III. Employment in academic institutions

- 2004–present: Assistant Professor at Institute of Mathematics of the University of Gdańsk.

- October 2007 – September 2008: postdoctoral fellowship at Ben-Gurion University of the Negev in Israel.

- September 2002 – December 2002: predoctoral fellowship at Fields Institute for Research in Mathematical Sciences in Canada.

IV. Scientific achievement (in the sense of Article 16 Paragraph 2 of the Act of 14 March 2003 on Academic Degrees and Title and on Degrees and Title in the Field of Art)

The series of 6 publications under the title

"Ideal convergence and infinite combinatorics"

consisting of the following publications:

[H1] R. Filipów, N. Mrożek, I. Recław, and P. Szuca, *Ideal convergence of bounded sequences*, J. Symbolic Logic **72** (2007), no. 2, 501–512.

[H2] R. Filipów and P. Szuca, *On some questions of Drewnowski and Łuczak concerning submeasures on* $\mathbb{N}$, J. Math. Anal. Appl. **371** (2010), no. 2, 655–660.

[H3] R. Filipów and P. Szuca, *Rearrangement of conditionally convergent series on a small set*, J. Math. Anal. Appl. **362** (2010), no. 1, 64–71.

[H4] R. Filipów and P. Szuca, *Three kinds of convergence and the associated* $\mathcal{I}$*-Baire classes*, J. Math. Anal. Appl. **391** (2012), no. 1, 1–9.

[H5] R. Filipów, *On Hindman spaces and the Bolzano-Weierstrass property*, Topology Appl. **160** (2013), no. 15, 2003–2011.

[H6] R. Filipów, *The reaping and splitting numbers of nice ideals*, Colloq. Math. **134** (2014), no. 2, 179–192.

What follows is the discussion of the results of the above-mentioned publications. Throughout the presentation the articles cited as [H1]–[H6] are related to the above-mentioned series of 6 publications; articles cited as [D1]–[D5] and [P1]–[P11] are related to my other publications with the full list at page 11 of the presentation; and the papers cited as [1]–[∞] are related to the publications of other authors with the full list at page 18.

Filipów

1. Introduction

In the above-mentioned publications we consider ideal convergence of sequences of points and functions and we study it with the aid of infinite combinatorics. A family $\mathcal{I}$ of subsets of $\mathbb{N}$ is called an *ideal* if it is closed under taking subsets and finite unions of its elements. We say that a sequence (x_n) in a topological space X is *$\mathcal{I}$-convergent* (or *ideal convergent* if the ideal $\mathcal{I}$ is clear from context) to $x \in X$ if for every neighborhood U of x there exists a set $A \in \mathcal{I}$ such that $x_n \in U$ for every $n \in \mathbb{N} \setminus A$. In the case of the ideal of sets of density zero the ideal convergence is known as the statistical convergence and it was already considered by Steinhaus [102, p. 73] and Schoenberg [96]. For P-ideals, the ideal convergence is equivalent to the existence of a set from the dual filter such that the subsequence defined on this set is ordinary convergent (see e.g. [62]). The notion of ideal convergence can be also describe in terms of "convergence of a filter to a point" introduced in 1937 by Cartan [24].

In 1991 Mazur [78] characterized all F_σ ideals in terms of lower semicontinuous submeasures on $\mathbb{N}$, and in 1999 Solecki [100] obtained a similar characterization for analytic P-ideals. Both characterizations provided us with new methods to examine ideals (see e.g. [38, 52]), as well as ideal convergence (see e.g. [P8]).

Another tool that helps to study ideal convergence (see e.g. [H4, 70, 87]) are infinite games defined by Laflamme [71] jointly with the theorem of Borel determinacy and some combinatorial characterizations of winning strategies in these games.

In the above-mentioned series of publications we use these methods to obtain some new results and in particular to solve some problems posted by Drewnowski and Łuczak [34], and Wilczyński [107]. Below, I show also how one can use our results to answer some questions posted later by Borodulin-Nadzieja, Farkas and Plebanek [17], and Hrušák [52].

2. Bolzano-Weierstrass property and nonatomic submeasures

In the paper [H1] we introduced the Bolzano-Weierstrass property (in short: BW property) for ideals. An ideal $\mathcal{I}$ has *BW property* if for every bounded sequence (x_n) of reals there is $A \notin \mathcal{I}$ such that the subsequence $(x_n)_{n\in A}$ is $\mathcal{I}$-convergent.

Using the notion of $\mathcal{I}$-small sets introduced by Farah [39], we proved [H1, Proposition 3.3] a characterization of ideals with BW property in terms of trees: an ideal $\mathcal{I}$ has BW property if and only if for every binary tree $\mathcal{T}$ of height ω such that every level of $\mathcal{T}$ is a partition of $\mathbb{N}$ there exists an infinite branch $\{B_n : n \in \mathbb{N}\}$ of $\mathcal{T}$ and a set $A \notin \mathcal{I}$ such that $A \setminus B_n \in \mathcal{I}$ for every n. Later, we repeatedly used this characterization to study BW property.

Using Mazur's characterization and the above characterization, we proved [H1, Proposition 3.4], that every F_σ ideal has BW property.

For analytic P-ideals, BW property turned out to be closely related to nonatomic submeasures on $\mathbb{N}$ (nonatomic submeasures are called "strongly continuous submeasures" in the book [15], and "compact submeasure" in the paper [89]). A submeasure ϕ is *nonatomic* if for every $\varepsilon > 0$ there is a finite partition $\mathbb{N} = A_1 \cup \cdots \cup A_n$ such that $\phi(A_i) < \varepsilon$ for all $i \leq n$.

Filip

By Solecki's characterization we know that for every analytic P-ideal $\mathcal{I}$ there exists a lower semicontinuous submeasure ϕ such that $\mathcal{I} = \{A : ||A||_\phi = 0\}$, where $||A||_\phi = \lim_n \phi(A \setminus \{0, 1, \ldots, n\})$.

In the paper [H1, Theorem 3.6] we proved that an analytic P-ideala $\mathcal{I}$ has BW property if and only if the submeasure $||\cdot||_\phi$ is not nonatomic. The main ingredients of the proof are Solecki's characterization and the above-mentioned characterization of BW property in terms of trees.

Nonatomic submeasure on $\mathbb{N}$ are considered in a series of papers by Drewnowski and Łuczak [33–35]. For instance, in [34] the authors find two necessary conditions for a submeasure to be equivalent to the lim sup of a sequence of lower semicontinuous submeasures and they pose two problems: (1) whether these conditions are equivalent to each other, and (2) if these conditions are also sufficient.

In the paper [H2] we solve both problems: (1) we proved that, in general, these conditions are not equivalent [H2, Theorem 5] (however for submeasures vanishing on singletons both conditions are equivalent [H2, Theorem 3]); (2) we constructed [H2, Example 8] a submeasure ϕ that satisfies both conditions but is not equivalent to the lim sup of a sequence of lower semicontinuous submeasures.

The construction of ϕ can be described in three steps. First, we prove that a submeasure ψ satisfies these conditions if and only if the family of nonzero sets of ψ is a P-coideal. Next, we show that if a submeasure ψ is equivalent to the lim sup of a sequence of lower semicontinuous submeasures, then the family of nonzero sets of ψ is a $G_{\delta\sigma}$ coideal. Finally, we use a maximal almost disjoint family to construct a P-coideal which is not $G_{\delta\sigma}$, and with the aid of this coideal we define ϕ.

In [34] the authors also find a necessary condition for a submeasure to be equivalent to the submeasure $||\cdot||_\phi$ where ϕ is a σ-submeasure (the authors call it the *core of a σ-submeasure*), and they pose a question if this condition is also sufficient.

In the paper [H2, Theorem 11] we prove that the answer to the question is negative. The proof can be split into three steps. First, we show that if the ideal of zero-sets of a submeasure ψ is a P-ideal, then ψ satisfies this necessary condition. Second, we prove that there is no σ-submeasure ϕ such that the coideal of nonzero-sets of $||\cdot||_\phi$ is a Q-coideal. Last, we take a selective ultrafilter (i.e. an ultrafilter which is a P-point and Q-point) and define the required submeasure. Certainly, for the last step we need some extra set-theoretic assumptions (e.g. the Continuum Hypothesis). We do not know if a submeasure with the above-mentioned properties exists in ZFC.

3. Katětov order on Borel ideals

The Katětov order was introduced by Katětov in [56]. Let $\mathcal{I}, \mathcal{J}$ be ideals on $\mathbb{N}$. Then $\mathcal{I}$ *is below* $\mathcal{J}$ *in the Katětov order* (in short: $\mathcal{I} \leq_K \mathcal{J}$) if there is a function $f : \mathbb{N} \to \mathbb{N}$ such that $f^{-1}[A] \in \mathcal{J}$ for all $A \in \mathcal{I}$.

The restriction of the Katětov order to maximal ideals coincides with the well-known Rudin-Keisler order. There also is a connection of the Katětov order with $\mathcal{I}$-ultrafilters introduced by Baumgartner in [13]. Namely, Flašková observed [42] that $\mathcal{U}$ is an $\mathcal{I}$-ultrafilter if and only if $\mathcal{I} \not\leq_K \mathcal{U}^*$. Many standard combinatorial properties of ultrafilters can be characterized in the same way with the aid of Borel ideals (see [43]). For instance, $\mathcal{U}$ is a P-point if and only if CONV $\not\leq_K \mathcal{U}^*$, where CONV is the ideal of all subsets of $\mathbb{Q}$ having only finitely many limit points.

Fdupt

There is an interesting result of Guzman-González and Meza-Alcántara [48] (a similar result was also obtained independently by Mrożek [83]) concerning the Katětov order restricted to Borel ideals which says that there are antichains of size $\mathfrak{c}$ and chains of size $\mathfrak{b}$ of Borel ideals in the Katětov order.

In [52, Question 5.16] Hrušák posed a question if a Borel ideal $\mathcal{I}$ can be extended to an F_σ ideal if and only if CONV $\not\leq_K \mathcal{I}$. Using a result of Meza-Alcántara [80], Hrušák's problem can be rephrased as a question whether an ideal $\mathcal{I}$ can be extended to an F_σ ideal if and only if $\mathcal{I}$ has finBW property (where finBW property is defined in the same manner as BW property but instead of ideal convergence we require the ordinary convergence of a subsequence [H1]).

In the paper [H1, Theorem 4.2] we proved that the answer to Hrušák's question is positive for Borel P-ideals (we do not know the answer for all Borel ideals). The main ingredient of the proof is the characterization of BW property in terms of nonatomic submeasures [H1, Theorem 3.6] which is discussed in Chapter 2 of the presentation.

4. Extending ideals to summable ideals

An ideal $\mathcal{I}$ is called a *summable ideal* if there is a divergent series $\sum a_n$ of non-negative reals such that $\mathcal{I} = \{A \subseteq \mathbb{N} : \sum_{n\in A} a_n < \infty\}$. Summable ideals were introduced by Mazur in [78], although some subclass of summable ideals was earlier considered by Mathias [74].

In 1930 Auerbach [3] proved that if a series of nonnegative reals $\sum_n a_n$ is divergent then there is a set $A \subseteq \mathbb{N}$ of density zero such that the subseries $\sum_{n\in A} a_n$ is also divergent (the same theorem was later repeatedly reproved, for instance by Agnew [1] in 1947 and by Estrad and Kanwal [37] in 1986). Using summable ideals, Auerbach's result can be rephrased in the following manner: the ideal of density zero sets cannot be extended to any summable ideal.

In [97], Freedman and Sember showed that the ideal of uniform density zero sets cannot be extended to any summable ideal (the summable ideals are called "full classes" in their paper).

Now we show that our results on BW property allow us to generalize the above theorems. Summable ideals have finBW property (it follows from the fact, that summable ideals are F_σ, and all F_σ ideals have finBW property [H1, Proposition 3.4]). On the other hand, if an ideal has finBW property, then every ideal contained in it also has finBW property. As a corollary we obtain that if an ideal does not have finBW property, then it cannot be extended to a summable ideal. Since the ideal of density zero sets and ideal of uniform density zero sets do not have finBW property (see [H1, P4]), so the above observation extends the results of Auerbach, Freedman and Sember.

Another result about extending ideals to summable ideals is a theorem of Paštéka [89] which says that no ideal of zero-sets of a nonatomic submeasure can be extended to a summable ideal. In [P4, Theorem 1] we proved that the ideal of zero-sets of a nonatomic submeasure does not have finBW property. This fact jointly with the earlier observation that ideals without finBW property cannot be extended to summable ideals can be regarded as a generalization of Paštéka's theorem.

Fdlpr

Two other papers about extending ideals to summable ideals, which are worth to mention, are papers [2,36] (in both papers the property that an ideal can be extended to a summable ideal is called "Positive Summability Property"). In [36] Drewnowski and Paúl proved among others that generalized density ideals (these ideals are connected with matrix summability methods) can be extended to a summable ideal if and only if they do not have the Nikodým property. In [2] Alon, Drewnowski and Łuczak using the Kneser graph constructed an ideal with the Nikodým property that has finBW property but cannot be extended to a summable ideal (in that paper the authors use the term "nonatomic ideals" instead of ideals without finBW property).

It turns out that the problem of extending ideals to summable ideals is connected with the Riemann rearrangement theorem (i.e. the theorem which says that if a series is conditionally convergent, then its terms can be rearranged so that the new series converges to any given value). In [107] Wilczyński strengthened the Riemann rearrangement theorem by proving that one can only rearrange terms whose indexes form a density zero set, and he posed a problem about characterization of all ideals having the same property, i.e. ideals $\mathcal{I}$ such that for every conditionally convergent series $\sum_n a_n$ and for every $r \in \mathbb{R}$ there exists a permutation $\sigma : \mathbb{N} \to \mathbb{N}$ such that $\sum_n a_{\sigma(n)} = r$ and $\{n : \sigma(n) \neq n\} \in \mathcal{I}$. We say that such ideals have the *Riemann property* (in short: *R property*).

In the paper [H3, Theorem 3.3] we solved this problem by proving that an ideal has the Riemann property if and only if it cannot be extended to a summable ideal. There are two main components of the proof of this characterization. First, we show that no summable ideal has R property. Second, assuming that an ideal $\mathcal{I}$ does not have R property, we use the series that witnesses to the lack of this property to construct a summable ideal extending $\mathcal{I}$.

In [17] Borodulin-Nadzieja, Farkas and Plebanek generalize the notion of summable ideals using series in Banach spaces or Polish groups, and call them ideals representable in a given space (summable ideals turn out to be ideals representable in the space $\mathbb{R}$). They proved that all analytic P-ideals are representable in Polish Abelian groups, and all nonpathological analytic P-ideals are representable in Banach spaces. Moreover, the authors posed a question [17, Question 6.13] about a characterization of ideals (analytic P-ideals) that can be extended to summable ideals.

Since the negation of the Riemann property characterizes ideals that can be extended to summable ideals, so it gives an answer to the above question for an arbitrary ideal. In the case of some subfamily of analytic P-ideals we can give another characterization. Namely, in the paper [H3, Proposition 3.7] we proved that for the class of all density ideals considered by Farah [38] (it is a proper subfamily of the family of all analytic P-ideals), R property is equivalent to the negation of BW property. As a corollary we obtain that a density ideal can be extended to a summable ideal if and only if it has BW property. However, this characterization does not hold for all analytic P-ideals, since we showed [H3, Example 3.6] that there exists an F_σ P-ideal (hence it has BW property) which cannot be extended to a summable ideal.

Fdyn

In [14] Bermúdez and Martinón used our characterization of ideals with R property to prove an ideal version of a theorem about changes of signs in a conditionally convergent series.

A vector-valued counterpart of the Riemann rearrangement theorem was proved by Lévy'ego and Steinitza at the beginning of the 20th century, and recently Klinga [58] proved that the ideal counterpart of Lévy-Steinitz theorem holds for ideals that cannot be extended to summable ideals.

5. Cardinal invariants of quotient Boolean algebras

A set $S \subseteq \mathbb{B}$ is a *splitting set* in a Boolean algebra $\mathbb{B}$ if for every nonzero element $b \in \mathbb{B}$ there exists $s \in S$ such that $b \cdot s \neq 0$ and $b - s \neq 0$.

It is known that any splitting set in the quotient Boolean algebra $\mathcal{P}(\mathbb{N})/\mathrm{Fin}$ (where Fin denotes the ideal of all finite sets) is uncountable. The minimal cardinality of a splitting set in $\mathcal{P}(\mathbb{N})/\mathrm{Fin}$ is denoted by $\mathfrak{s}$ and is called the *splitting number* (see e.g. [106] or [16]).

It turns out that there is a relationship between splitting sets and BW property. In the paper [H1, Theorem 5.1] we proved that an ideal $\mathcal{I}$ does not have BW property if and only if there exists a countable splitting set in $\mathcal{P}(\mathbb{N})/\mathcal{I}$.

The main ingredient of the proof of this theorem is the characterization of BW property in terms of trees (see Chapter 2 of the presentation). The proof can be briefly sketch in the following way. If $\mathcal{P}(\mathbb{N})/\mathcal{I}$ has a countable splitting set S, then we construct an appropriate tree (at a given level we split sets from the previous level with the aid of S) and using the characterization we show that $\mathcal{I}$ does not have BW property. On the other hand, if an ideal $\mathcal{I}$ does not have BW property, then we take a tree $\mathcal{T}$ from the characterization and show that the union of all levels of $\mathcal{T}$ is a countable splitting set in $\mathcal{P}(\mathbb{N})/\mathcal{I}$.

In [5, 50] Balcar, Hernández-Hernández and Hrušák proved that there is a countable splitting set in $\mathcal{P}(\mathbb{N})/\mathcal{I}_d$ (where $\mathcal{I}_d$ is the ideal of all sets of density zero) and in $\mathcal{P}(\mathbb{Q})/\mathrm{NWD}$ (where NWD is the ideal of all nowhere dense subsets of $\mathbb{Q}$). Since the ideals NWD and $\mathcal{I}_d$ do not have BW property ([H1]), so their result can be regarded as a particular case of our theorem.

In [50] the authors proved also that for an analytic P-ideal $\mathcal{I}$, if $\mathcal{P}(\mathbb{N})/\mathcal{I}$ has a countable splitting set, then the ideal $\mathcal{I}$ is totally bounded (i.e. $\phi(\mathbb{N}) < \infty$ for every submeasure ϕ that defines $\mathcal{I}$ according to Solecki's characterization). Now we show that this theorem also follows from our results about BW property. Using the characterization of BW property for analytic P-ideals in terms of nonatomic submeasures (see Chapter 2 of the presentation), we know that if an ideal $\mathcal{I}$ does not have BW property, then the submeasure $||\cdot||_\phi$ is nonatomic. Moreover, one can show that if $||\cdot||_\phi$ is nonatomic, then $\phi(\mathbb{N}) < \infty$. Thus, if $\mathcal{P}(\mathbb{N})/\mathcal{I}$ has a countable splitting set, then $\mathcal{I}$ does not have BW property, hence $\mathcal{I}$ is totally bounded.

It is consistent with the axioms of ZFC that $\mathcal{P}(\mathbb{N})/\mathrm{Fin}$ does not have a splitting set of cardinality less than $\mathfrak{c}$ (even if $\aleph_1 < \mathfrak{c}$). For instance, it holds under Martin's axiom and the negation of the Continuum Hypothesis (see e.g. [16]).

In the paper [H6, Theorem 4.3] we proved that the same result holds for every analytic P-ideal with BW property (recall that for an ideal without BW property there always is a countable splitting set).

Filip

Here is a brief sketch of the proof. We start with a family $\mathcal{F}$ of less than $\mathfrak{c}$ subsets of $\mathbb{N}$, and we will show that it is not a splitting set. With the aid of $\mathcal{F}$ we define a partially ordered set $\mathbb{P}$ (in this case it is the so-called Mathias poset with respect to $\mathcal{F}$) and an appropriate family $\mathcal{D}$ of dense subsets of $\mathbb{P}$. Then, by Martin's axiom, there is a filter $F \subseteq \mathbb{P}$ that intersects all elements of $\mathcal{D}$. Using the filter F we define a set that is not split by any member of $\mathcal{F}$ (thus $\mathcal{F}$ is not a splitting set).

In this outline, there are two steps for which we need some additional tools to carry out. First, we use Solecki's characterization of analityc P-ideals to define an appropriate dense sets (we use the fact that considered submeasures are lower semicontinuous, hence the sets from outside the ideal can be approximate by finite sets). Second, we need the lemma [H6, Lemma 2.7] to show that sets from $\mathcal{D}$ are dense. Actually the lemma gives another characterization of BW property, namely an analytic P-ideal has BW property if and only if there is $\delta > 0$ and $x : \mathcal{P}(\mathbb{N}) \to \{0,1\}$ such that $\|\bigcap\{A^{x(A)} : A \in \mathcal{G}\}\|_\phi > \delta$ for every finite family $\mathcal{G} \subseteq \mathcal{P}(\mathbb{N})$ (where $A^0 = A$ and $A^1 = \mathbb{N} \setminus A$). It is worth to notice that the function x does not depend on a choice of a family $\mathcal{G}$ i.e. for every set A we know in advance if we take A or $\mathbb{N} \setminus A$ in the intersection for every family $\mathcal{G}$ containing A.

In [H6, Theorem 3.2] we prove also that Martin's axiom implies that there is no splitting set of cardinality less than $\mathfrak{c}$ in $\mathcal{P}(\mathbb{N})/\mathcal{I}$ for every F_σ ideal. The same results for F_σ ideals and analytic P-ideals was obtained independently by Mrożek [83].

In the paper [H6] we study also the *reaping number* $\mathfrak{r}$. We proved that in $\mathcal{P}(\mathbb{N})/\mathcal{I}$, where $\mathcal{I}$ is an F_σ ideal or an analytic P-ideal, the reaping number is uncountable [H6, Proposition 3.1 and 4.1], and under Martin's axiom the reaping number equals $\mathfrak{c}$ [H6, Theorem 3.2 and 4.2]. It is worth to notice, that in this case the results hold for all ideals (with and without BW property). Another interesting result about the number $\mathfrak{r}$ was obtained by Steprāns [103] who proved that the reaping number in $\mathcal{P}(\mathbb{N})/\mathcal{I}_d$ (where $\mathcal{I}_d$ is the ideal of all density zero sets) equals $\mathfrak{c}$ in ZFC.

Another cardinal invariant we are interested in is the *bounding number* $\mathfrak{b}$. This number in quotient Boolean algebras $\mathcal{P}(\mathbb{N})/\mathcal{I}$ was already considered by Farkas and Soukup [40], and Brendle and Mejía [19]. For instance, in [40] the authors proved that the number $\mathfrak{b}$ in $\mathcal{P}(\mathbb{N})/\mathcal{I}$, where $\mathcal{I}$ is an analytic P-ideal, is equal to its classical counterpart in $\mathcal{P}(\mathbb{N})/\mathrm{Fin}$. Careful reading of their proof jointly with Talagrand's characterization [104] of ideals with the Baire property reveales that their result can be extended to all ideals with the Baire property.

The bounding number turned out to be important in comparing ideal pointwise convergence with ideal equal convergence of sequences of functions. This topic is discussed in detail at page 15 of the presentation.

6. Equal (quasi-normal) convergence of sequences of continuous functions

A sequence of functions $f_n : X \to \mathbb{R}$ is *ideal pointwise convergent* to $f : X \to \mathbb{R}$ if the sequence of reals $(f_n(x))_n$ is ideal convergent to $f(x)$ for every $x \in X$.

Ideal pointwise convergence was already considered in 1970s by Katětov [55, 57]. He proved, among others, that for every countable ordinal α there exists a Borel ideal such that the family of all ideal pointwise limits of sequences of continuous functions equals the family of all functions of Baire class α (in particular, there is an ideal and a sequence of continuous functions such that the ideal pointwise limit of the sequence is not of Baire class one).

It is also known that for every maximal ideal there is a sequence of continuous functions such that the ideal pointwise limit of the sequence is not even Lebesgue measurable.

On the other hand, it seems to be interesting to describe ideals for which ideal pointwise limits of sequences of continuous functions are of Baire class one. In [62] Kostyrko, Šalát and Wilczyński proved that the ideal of density zero sets has this property. Later, the problem was studied by Laczkovich and Recław [70] and Debs and Saint Raymond [31]. For instance, in [70] and independently in [31] the authors proved that ideal pointwise limits of sequences of continuous functions are of Baire class one if and only if the ideal does not contain an isomorphic copy of the ideal $\mathrm{Fin} \times \mathrm{Fin}$.

In the paper [H4], beside pointwise convergence, we consider also discrete convergence and equal convergence (also known as quasi-normal convergence or σ-uniform convergence). Both kinds of convergence were introduced by Császár and Laczkovich [27] (discrete convergence was also considered by Sierpiński [99] in the case of transfinite sequences). Topological spaces defined with the aid of equal convergence (the so-called QN-spaces) were examined by Bukovský, Recław and Repický in the series of papers [22, 23, 94].

In the paper [H4] we use an infinite game of Laflamme [71] to describe ideal (pointwise, discrete and equal) Baire classes. Below I describe the results we obtained for equal convergence.

A sequence of functions $f_n : X \to \mathbb{R}$ is *ideal equal convergent* to $f : X \to \mathbb{R}$ if there is a sequence of nonnegative reals $(\varepsilon_n) \to 0$ such that $\{n : |f_n(x) - f(x)| \geq \varepsilon_n\} \in \mathcal{I}$ for every $x \in X$.

Let $\mathcal{I}$ be an ideal. Let us consider the following game $G(\mathcal{I})$. Player I in the nth move plays a set $A_n \in \mathcal{I}$, and then player II plays a finite set $F_n \subseteq \mathbb{N} \setminus A_n$. Player I wins when $\bigcup_n F_n \in \mathcal{I}$. Otherwise player II wins.

We proved [H4, Theorem 5.5] that if player II has a wining strategy in the game $G(\mathcal{I})$, then ideal equal limits of sequences of functions of equal Baire class α are of ordinary equal Baire class $\alpha + 1$ for every finite α (in particular, ideal equal limits of sequences of continuous functions are of ordinary equal Baire class one). The problem for infinite α remains open.

There are two main ingredients of the proof (for simplicity, I outline only the case of sequences of continuous functions). First, we use Laflamme's characterization [71] of ideals for which player II has a winning strategy. The characterization is rather technical and I will not provide it here, but the most important feature of this characterization is that all sets from the dual filter can be described by a fixed countably family of finite sets. Second, we use Császár-Laczkovich characterization [28] of functions of equal Baire class one: f is of equal Baire class one if and only if there are closed sets $A_n \subseteq X$ such that $\bigcup_n A_n = X$ and $f \restriction A_n$ can be extended to a continuous function on X for every n.

In [87] Natkaniec and Szuca use another Laflamme's game to study ideal pointwise limits of sequences of quasi-continuous functions. Ideal discrete and equal

Fly

limits of sequences of quasi-continuous functions are studed by Kwela, Natkaniec, Staniszewski and Szuca, and the papers with their results are in preparation.

We continue the study of various kinds of ideal convergence of sequences of functions and the results we have obtained so far are published in the papers [P10, P11] which are described in detail at page 15 of the presentation.

7. Hindman spaces

A topological space X has *BW property with respect to an ideal* $\mathcal{I}$ if every sequence in X has $\mathcal{I}$-convergent subsequence defined on a set from outside the ideal ([H1]).

One can see that an ideal $\mathcal{I}$ has BW property if and only if the interval $[0,1]$ has BW property with respect to $\mathcal{I}$. If a Hausdorff topological space X has BW property with respect to some ideal, then X is countably compact. A space X has BW property with respect to the ideal of all finite subsets of $\mathbb{N}$ if and only if X is sequentially compact. Moreover, every metric compact space has BW property with respect to every ideal with BW property (this theorem is not true if we drop the assumption about metrizability) [H1, p. 505].

A set $A \subseteq \mathbb{N}$ is an *IP-set* if there is an infinite $B \subseteq \mathbb{N}$ such that $\mathrm{FS}(B) \subseteq A$ (where $\mathrm{FS}(B)$ is the set of all finite sums of elements of B). Hindman's theorem [51] says that for every finite partition $\mathbb{N} = A_1 \cup \cdots \cup A_n$ there is $i \leq n$ such that A_i is an IP-set.

Topological spaces connected with Hindman's theorem were studied in a series of papers by Kojman, Jones and Shelah [53, 59, 61]. The definition of these spaces requires the notion of IP-convergence introduced by Furstenberg and Weiss [47]. A subsequence $(x_n)_{n\in\mathrm{FS}(A)}$ is *IP-convergent to* x if for every neighborhood U of x there is $N \in \mathbb{N}$ such that $x_n \in U$ for every $n \in \mathrm{FS}(A \setminus \{0, 1, \dots, N\})$. A topological space X is a *Hindman space* if every sequence in X has an IP-convergent subsequence ([59]).

In [41] Flašková compares Hindman spaces to spaces having BW property with respect to summable ideals. Among others, she proved that there is a space having BW property with respect to a given summable ideal that is not a Hindman space.

In the paper [H5] we study relationship between Hindman spaces and spaces with BW property with respect to *Hindman ideal* $\mathcal{H} = \{A \subseteq \mathbb{N} : A \text{ is not an IP-set}\}$.

We proved [H5, Proposition 3.5] that every Hindman space has BW property with respect to $\mathcal{H}$. On the other hand, we constructed [H5, Theorem 3.7] a topological space that has BW property with respect to $\mathcal{H}$ but is not a Hindman space. The example we provided is a one-point compactification of the Mrówka space [81] defined with the aid of maximal almost disjoint family of sets from $\mathcal{H}$.

In the same paper we proved [H5, Theorem 3.3] that Hindman ideal $\mathcal{H}$ has BW property. In the proof of this theorem we use idempotent ultrafilters on $\mathbb{N}$ i.e. ultrafilters $\mathcal{U}$ such that $\mathcal{U} + \mathcal{U} = \mathcal{U}$ where "$+$" is an extension of addition of natural numbers on the family of all ultrafilters. The second tool we use in the proof is the characterization of BW property in terms of trees [H1, Proposition 3.3] (see Chapter 2 of the presentation for details).

We continue the study of topological spaces with BW property with respect to ideals and the results we have obtained so far are published in the paper [P9] which is described in detail at page 16 of the presentation.

Flr

V. Description of the remaining scientific achievements

The remaining scientific achievements consist of the following publications.

Publications containing results obtained before PhD:

[D1] R. Filipów and I. Recław, *On the difference property of Borel measurable and (s)-measurable functions*, Acta Math. Hungar. **96** (2002), no. 1-2, 21–25.

[D2] R. Filipów, *On the difference property of the family of functions with the Baire property*, Acta Math. Hungar. **100** (2003), no. 1-2, 97–104.

[D3] R. Filipów, *On the difference property of families of measurable functions*, Colloq. Math. **97** (2003), no. 2, 169–180.

[D4] F. G. Dorais and R. Filipów, *Algebraic sums of sets in Marczewski-Burstin algebras*, Real Anal. Exchange **31** (2005/06), no. 1, 133–142.

[D5] F. G. Dorais, R. Filipów, and T. Natkaniec, *On some properties of Hamel bases and their applications to Marczewski measurable functions*, Cent. Eur. J. Math. **11** (2013), no. 3, 487–508.

Publications containing results obtained after PhD:

[P1] R. Filipów and P. Szuca, *Density versions of Schur's theorem for ideals generated by submeasures*, J. Combin. Theory Ser. A **117** (2010), no. 7, 943–956.

[P2] R. Filipów, N. Mrożek, I. Recław, and P. Szuca, *Ideal version of Ramsey's theorem*, Czechoslovak Math. J. **61(136)** (2011), no. 2, 289–308.

[P3] R. Filipów, A. Nowik, and P. Szuca, *There are measurable Hamel functions*, Real Anal. Exchange **36** (2010/11), no. 1, 223–229.

[P4] P. Barbarski, R. Filipów, N. Mrożek, and P. Szuca, *Uniform density u and $\mathcal{I}_u$-convergence on a big set*, Math. Commun. **16** (2011), no. 1, 125–130.

[P5] R. Filipów, N. Mrożek, I. Recław, and P. Szuca, *$\mathcal{I}$-selection principles for sequences of functions*, J. Math. Anal. Appl. **396** (2012), no. 2, 680–688.

[P6] R. Filipów, N. Mrożek, I. Recław, and P. Szuca, *Extending the ideal of nowhere dense subsets of rationals to a P-ideal*, Comment. Math. Univ. Carolin. **54** (2013), no. 3, 429–435.

[P7] P. Barbarski, R. Filipów, N. Mrożek, and P. Szuca, *When does the Katětov order imply that one ideal extends the other?*, Colloq. Math. **130** (2013), no. 1, 91–102.

Filip

[P8] R. Filipów, T. Natkaniec, and P. Szuca, *Ideal convergence*, Traditional and present-day topics in real analysis, Faculty of Mathematics and Computer Science. University of Łódź, Łódź, 2013, pp. 69–91.

[P9] R. Filipów and J. Tryba, *Convergence in van der Waerden and Hindman spaces*, Topology Appl. **178** (2014), 438–452.

[P10] R. Filipów and M. Staniszewski, *On ideal equal convergence*, Cent. Eur. J. Math. **12** (2014), no. 6, 896–910.

[P11] R. Filipów and M. Staniszewski, *Pointwise versus equal (quasi-normal) convergence via ideals*, J. Math. Anal. Appl. **422** (2015), no. 2, 995–1006.

What follows is the discussion of the results of the above-mentioned publications.

The difference property in de Bruijn sense. In the papers [D1, D2, D3] we study the difference property in de Bruijn sens (in short: the difference property) for families of measurable functions. A family $\mathcal{F}$ of real-valued functions defined on $\mathbb{R}$ has the *difference property* if for any function f such that $\Delta_h f \in \mathcal{F}$ for every $h \in \mathbb{R}$ (where $\Delta_h f(x) = f(x+h) - f(x)$) there is a function $g \in \mathcal{F}$ and an additive function $A : \mathbb{R} \to \mathbb{R}$ such that $f = g + A$. This notion was introduced by de Bruijn [30] in 1951. Since then the difference property has been examined for many classes of functions and some generalizations of the notion have been considered. A survey paper by Laczkovich [68] is a very good source of information on the notion.

Erdős, assuming the Continuum Hypothesis, proved (see e.g. [68]) that the family of Lebesgue measurable functions (the family of functions with the Baire property, respectively) does not have the difference property. On the other hand, Laczkovich proved [69] that it is consistent with the axioms of ZFC that the family of Lebesgue measurable functions has the difference property, and in the paper [68] he posed a problem if it consistent with ZFC that the family of functions with the Baire property has the difference property.

In the paper [D2, Theorem 1.2] we proved that the answer to the above problem is positive. In the paper [75] Matrai also solved the same problem using a different technique.

In the paper [D1, Theorem 2.2] we proved that the family of Marczewski measurable functions (a.k.a. (s)-measurable functions) does not have the difference property in ZFC.

In the paper [67] Laczkovich posed the following problem. Suppose that functions $\Delta_h f$ are Borel for all $h \in \mathbb{R}$. Is it true that they are of bounded Borel class?

In the paper [D1, Theorem 3.1] we proved that if we assume the Continuum Hypothesis then the answer to the above problem is negative. The same answer was also obtain by Ciesielski and Pawlikowski in the paper [25] under the assumption of the axiom CPA (Covering Property Axiom). Other results connected with this problem were obtained later by Fujita and Mátrai in the papers [45, 46].

Filipów

Nonmeasurable algebraic sums in Marczewski-Burstin algebras. An algebra of sets $\mathcal{A} \subseteq \mathcal{P}(X)$ is a *Marczewski-Burstin algebra generated by a family* $\mathcal{F}$ if $\mathcal{A} = \{A \subseteq X : (\forall F \in \mathcal{F})(\exists G \in \mathcal{F})(G \subseteq F \cap A \vee G \subseteq F \setminus A)\}$. The notion of a Marczewski-Burstin algebra generated by a family $\mathcal{F}$ (in short: MB algebra) was introduced by Brown and Elalaoui-Talibi in the paper [20], and later it was studied for instance by Balcerzak, Bartoszewicz, Ciesielski and Koszmider in the papers [6–8, 10–12]. The families of Lebesgue measurable sets, sets with the Baire property and (s)-measurable sets are examples of MB algebras.

Sierpiński [98] proved that there is a Lebesgue null set A such that the algebraic sum $A + A = \{x + y : x, y \in A\}$ is not Lebesgue measurable.

In the paper [D4] we study algebraic sums of null sets in MB algebras. We provide [D4, Theorem 7] a sufficient condition on the generating family of MB algebra to obtain a null set $A \in \mathcal{A}$ such that $A + A \notin \mathcal{A}$. To prove this theorem we construct [D4, Theorem 6] an almost invariant null set in $\mathcal{A}$.

Then we applied this theorem to obtain (s_0)-sets and Miller null sets whose algebraic sums are not measurable in the appropriate sense. These results were also obtained independently by Kysiak [66].

Discontinuous but measurable additive functions. It is known that if an additive function is discontinuous then it is Lebesgue nonmeasurable (see e.g. [63]).

In the paper [D5, Theorem 4.2] we proved that it is consistent with ZFC that there exists a discontinuous additive function which is (s)-measurable (we even proved that there is a discontinuous linear isomorphism of the vector spaces $\mathbb{R}^n$ and $\mathbb{R}^k$ over the field $\mathbb{Q}$ of rational numbers which is (s)-measurable [D5, Theorem 4.7]).

The above-mentioned functions are constructed under two distinct set-theoretical assumptions. In the first construction we assume that $\mathrm{cov}(\mathcal{M}) = \mathfrak{c}$, and in the second case we use the axiom CPA (Covering Property Axiom) introduced by Ciesielski and Pawlikowski [26]. In both constructions the most important ingredient is the existence of some "nice" Hamel basis (i.e. a basis of the space $\mathbb{R}$ over the field $\mathbb{Q}$). We proved [D5, Theorem 3.19] that such a Hamel basis exists under $\mathrm{cov}(\mathcal{M}) = \mathfrak{c}$, and it was constructed by Ciesielski and Pawlikowski [25] under CPA. To prove theorem [D5, Theorem 3.19] we generalized [D5, Theorem 2.3] Mycielski's theorem [84] about independent sets in relational systems.

Measurable functions whose graphs are Hamel bases. A function $f : \mathbb{R} \to \mathbb{R}$ is a *Hamel function* if the graph of f is a Hamel basis of the vector space $\mathbb{R}^2$ over the field $\mathbb{Q}$.

Hamel functions were introduced by Płotka in [90]. Together with Recław, he studied properties of these functions in the papers [91–93]. They proved, among others, that there exists a Hamel function which can be covered by finitely many partial continuous functions (it can be shown that there is no continuous Hamel function).

Hamel function were also examined by Matusik and Natkaniec in the series of papers [76, 77, 85, 86]. The authors proved, among others, that there are quasi-continuous Hamel functions and there are no approximately continuous Hamel functions.

flu

In the paper [90] Płotka proved that every function $f : \mathbb{R} \to \mathbb{R}$ is a sum of two Hamel function. As a corollary we obtain that there are Hamel functions which are Lebesgue nonmeasurable, do not have the Baire property or are not (s)-measurable. Moreover, it can be shown that no Borel function is a Hamel function.

In the paper [P3, Theorem 1 i 2] we proved that there are Hamel functions which are Lebesgue measurable, have the Baire property or are (s)-measurable.

Iterated versions of Ramsey and Schur's theorems. In the paper [44] Frankl, Graham and Rödl proved an iterated version of Ramsey's theorem which says that for every finite coloring $[\mathbb{N}]^2 = C_1 \cup \cdots \cup C_r$ of pairs of natural numbers with r colors there exists $\delta = \delta(r) > 0$ (which depends on the number of colors) such that for some $i \leq r$ we have

$$\overline{d}\Big(\Big\{x : \overline{d}\Big(\Big\{y : \overline{d}(\{z : \{x,y\},\{y,z\},\{x,z\} \in C_i\}) \geq \delta\Big\}\Big) \geq \delta\Big\}\Big) \geq \delta,$$

where $\overline{d}$ denotes the upper density.

In the paper [P1, Theorem 3.1] we proved that instead of the upper density in the above theorem one can put any submeasure of the form $||A||_\phi = \limsup_{n\to\infty} \phi(A \setminus \{0,1,\dots,n\})$, where ϕ is a lower semicontinuous submeasure (one can show that the upper density $\overline{d}$ is of that form). Submeasures of the form $||\cdot||_\phi$ are important since all analytic P-ideals are characterized by Solecki as ideals of zero sets for such submeasures.

Moreover, in the paper [P2, Corollary 5.7] we proved that if a submeasure $||\cdot||_\phi$ is not nonatomic, then the number δ in the above theorem does not depend on the number of colors.

In the paper [P2] we also consider some non-iterated versions of Ramsey's theorem. For instance, we proved [P2, Theorem 4.3] that an ideal $\mathcal{I}$ has the Bolzano-Weierstrass property if and only if for any finite coloring $[\mathbb{N}]^2 = C_1 \cup \cdots \cup C_r$ there is a set $A \notin \mathcal{I}$ which is $\mathcal{I}$-monochromatic (i.e. there is $k \leq r$ such that for every $a \in A$ we have $\{b \in A : \{a,b\} \notin C_k\} \in \mathcal{I}$).

Our study of the above Ramsey-like properties of ideals was continued by Meza-Alcántara [80] and Kwela [64,65] (among others, the authors solved some problems poseted in our paper [P2]).

The ordinary Schur's theorem says that for any finite coloring of natural numbers there are two numbers x, y such that x, y and $x + y$ have the same color. In the paper [44] Frankl, Graham and Rödl proved some iterated version of Schur's theorem which says that for any coloring $\mathbb{N} = C_1 \cup \cdots \cup C_r$ of natural numbers with r colors there is $\delta = \delta(r) > 0$ (which depends on the number of colors) such that for some $i \leq r$ we have

$$\overline{d}\Big(\Big\{x : \overline{d}(\{y : x, y, x + y \in C_i\}) \geq \delta\Big\}\Big) \geq \delta.$$

In the paper [P1] we consider iterated version of Schur's theorem for submeasures and we obtained similar results as in the case of iterated version of Ramsey's theorem: [P1, Theorem 3.2] – for any submeasure, [P1, Theorem 4.5] – for not nonatomic submeasures and independence of δ on the number of colors. The only difference compare with Ramsey's theorem is that we have to require that submeasures $||\cdot||_\phi$ are invariant under translations.

Various kinds of ideal convergence of sequences of functions. In the papers [P10, P11] we study relationships between various kinds of ideal convergence (e.g. pointwise, equal, uniform and σ-uniform) of sequences of real-valued functions.

This sort of research was started by Das, Dutta and Pal in the paper [29], where the authors proved some relationships between these kind of convergence and posted some problems. We solved these problems in the paper [P10, Example 4.7, Corollary 5.4, Corollary 6.5]. For instance, we showed that there exits ideal pointwise convergent sequence of function which is not ideal equal convergent; and we proved that ideal equal convergence implies ideal σ-uniform convergence if and only if the ideal is countably generated.

In the paper [P11] we focus only on ideal pointwise and equal convergence. For instance, we proved [P11, Theorem 5.3] that ideal pointwise convergence implies ideal equal convergence if and only if the domain of considered functions is not "too big" in the sense of cardinality. The notion of being not "too big" we expressed in terms of some cardinal invariant. In the ordinary case (i.e. for the ideal of finite sets) this cardinal invariant coincides with the bounding number $\mathfrak{b}$.

Currently, we keep research on this cardinal invariant. So far we have proved that this invariant equals $\mathfrak{b}$ for all P-ideals with the Baire property. For non-P-ideals we have had only some partial results and we still lack a result that encompasses a large class of ideals.

Relationships between ideal pointwise and equal convergence is also considered in a recent paper of Bukovský, Das and Šupina [21].

Ideal version of Arzela-Ascoli, Mazurkiewicz and Helly's theorems. To every theorem about convergence of sequences one can ask a question whether this theorem remains true when the term "convergence" is replaced with "ideal convergence". One can even ask a more general question about a characterization of ideals for which this "idealized" theorem holds. For instance, we did so in the case of "idealized" version of Bolzano-Weierstrass theorem (see Chapter 2 of Part IV of the presentation).

Problems of this sort for sequences of functions were already considered in the literature. For instance, Balcerzak, Dems and Komisarski [9] proved that the ideal version of Egoroff's theorem holds for the ideal of density zero sets, and Mrożek [82] generalized this result on all analytic P-ideals (another results concerning Egoroff's theorem can be found in [54]). Another example is connected with ideal version of Fatou's lemma which was considered by Louveau [72], and Solecki [101] characterized universally measurable ideals for which ideal version of the lemma holds.

In the paper [P5] we study ideal versions of three theorems concerning convergence of sequences of functions: Arzela-Ascoli theorem, Mazurkiewicz's theorem and Helly's theorem.

In the case of the first theorem we proved a characterization [P5, Theorem 3.1] which says that ideal version of Arzela-Ascoli holds if and only if the ideal has BW property.

In the paper [79] Mazurkiewicz proved that for any uniformly bounded sequence of continuous functions $f_n : X \to \mathbb{R}$ there is a perfect set $P \subseteq X$ and an infinite $A \subseteq \mathbb{N}$ such that the subsequence $(f_n \restriction P)_{n\in A}$ is uniformly convergent. In this case

Elp

we proved [P5, Theorem 4.1] that ideal version of this theorem holds for every ideal that can be extended to an F_σ ideal (the problem of extending ideals to F_σ ideals is described in Chapter 3 of Part IV of the presentation).

One can show that every ideal extending to an F_σ ideal has BW property, and ideal version of Mazurkiewicz's theorem does not hold for any ideal without BW property. However, we do not know if BW property characterizes ideals for which ideal version of Mazurkiewicz's theorem holds [P5, Problem 4.3].

Helly's theorem [49] says that any uniformly bounded sequence of monotone functions has a poinwise convergent subsequence. In this case we also proved [P5, Theorem 5.8] that ideal version of this theorem holds for every ideal that can be extended to an F_σ ideal and we still do not know if BW property characterizes ideals for which ideal version of Helly's theorem holds [P5, Problem 5.10]. In the case of van der Waerden ideal (the definition of this ideal is below), ideal version of Helly's theorem was earlier proved by Kojman [60].

Hindman and van der Waerden spaces. The paper [P9] is a continuation of the paper [H5]. In this paper, beside Hindman spaces, we also study van der Waerden spaces introduced by Kojman [60].

Van der Waerden's theorem [105] says that for every finite partition $\mathbb{N} = A_1 \cup \dots \cup A_n$ there is $i \leq n$ such that A_i contains arbitrarily long finite arithmetic progressions. *Van der Waerden ideal* is a family of such subsets $A \subseteq \mathbb{N}$ for which van der Waerden's theorem does not hold if we replace $\mathbb{N}$ with A.

A topological space X has *hBW property with respect to an ideal* $\mathcal{I}$ if every sequence $(x_n)_{n\in A}$ in X with $A \notin \mathcal{I}$ has $\mathcal{I}$-convergent subsequence defined on a set from outside the ideal ([H1]).

Two main results from the paper [P9, Theorems 3.3 and 4.5] say that BW property and hBW property with respect to Hindman (van der Waerden, resp.) ideal are equivalent for any topological space.

Katětov order. The Katětov order $\leq_K$ (see Chapter 3 of Part IV of the presentation for the definition) was used by Katětov in the paper [57] to study ideal limits of sequences of continuous functions.

Laczkovich and Recław [70] and independently Debs and Saint Raymond [31] proved that ideal limits of sequences of continuous functions are of the ordinary Baire class one if and only if the ideal $\mathcal{I}$ does not contain an isomorphic copy of the ideal $\mathrm{Fin} \times \mathrm{Fin}$ (in short: $\mathrm{Fin} \times \mathrm{Fin} \not\sqsubseteq \mathcal{I}$).

In the paper [H4, Theorem 6.2] we proved that $\mathrm{Fin} \times \mathrm{Fin} \not\sqsubseteq \mathcal{I}$ if and only if $\mathrm{Fin} \times \mathrm{Fin} \not\leq_K \mathcal{I}$ for every ideal $\mathcal{I}$. This result was the starting point to study other ideals $\mathcal{J}$ (beside $\mathrm{Fin} \times \mathrm{Fin}$) that have a similar property: $\mathcal{J} \leq_K \mathcal{I} \iff \mathcal{J} \sqsubseteq \mathcal{I}$ for every ideal $\mathcal{I}$. In this case we say that the ideal $\mathcal{J}$ has the *Katětov property.*

In the paper [P7, Theorem 3.4] we proved a characterization of the Katětov property: an ideal $\mathcal{J}$ has the Katětov property if and only if $\mathcal{J} \sqsubseteq \mathcal{J} \times \emptyset$. Moreover we showed [P7, Proposition 3.6] that if an ideal has the Katětov property then it is a local Q-ideal, and we proved [P7, Proposition 3.10] that the Katětov property is closed under the Fubini products of ideals.

Flyn

Extending the ideal NWD to a P-ideal. In the paper [32] Dow proved that assuming the Continuum Hypothesis the ideal NWD of nowhere dense subsets of $\mathbb{Q}$ can be extended to a P-ideal.

In the paper [P6, Corollary 1.3 and 2.2] we proved that the above extension cannot be "too nice" (namely the ideal NWD cannot be extended to any analytic P-ideal) and it cannot be "too large" (namely NWD cannot be extended to a maximal P-ideal).

The ideal of uniform density zero sets. In the paper [P4] we study the ideal $\mathcal{I}_u$ of sets of uniform density zero. The starting point for this paper was an observation that in the paper of Baláz and Šalát [4] the authors assumed wrongly that $\mathcal{I}_u$ is a P-ideal.

We also proved [P4, Theorem 1] that if a submeasure is nonatomic then the ideal of zero sets does not have BW property. Then, using this result, we showed that the ideal $\mathcal{I}_u$ does not have BW property. Moreover we proved [P4, Theorem 2] that the ideal $\mathcal{I}_u$ is $F_{\sigma\delta}$ and is not $G_{\delta\sigma}$.

Last but not least. The paper [P8] is a chapter in the monograph *„Traditional and present-day topics in real analysis"* dedicated to Professor Jan Lipiński on the occasion of his 90th birthday. Our chapter is a survey that focuses on three aspects of ideal convergence. In the first part we consider ideal convergence of bounded sequences (in other words this part is devoted to ideals with the Bolzano-Weierstrass property). The second part deals with ideal convergence of sequences of continuous functions (i.e. we consider ideal Baire classes of functions). The last part contains results about the set of points where a sequence of functions is convergent (i.e. we consider ideal versions of the so-called Lunina's 7-tuples [73]). The research on ideal version of Lunina's 7-tuples was started by Borzestowski and Recław [18], and it was continued by Recław [95] and Natkaniec and Wesołowska [88].

Fly

References

[1] R. P. Agnew, *Subseries of series which are not absolutely convergent*, Bull. Amer. Math. Soc. **53** (1947), 118–120.

[2] N. Alon, L. Drewnowski, and T. Łuczak, *Stable Kneser hypergraphs and ideals in $\mathbb{N}$ with the Nikodým property*, Proc. Amer. Math. Soc. **137** (2009), no. 2, 467–471.

[3] H. Auerbach, *Über die Vorzeichenverteilung in unendlichen Reihen.*, Studia Math. **2** (1930), 228–230.

[4] V. Baláž and T. Šalát, *Uniform density u and corresponding I_u-convergence*, Math. Commun. **11** (2006), no. 1, 1–7.

[5] B. Balcar, F. Hernández-Hernández, and M. Hrušák, *Combinatorics of dense subsets of the rationals*, Fund. Math. **183** (2004), no. 1, 59–80.

[6] M. Balcerzak and J. Rzepecka, *On Marczewski-Burstin representations of algebras and ideals*, J. Appl. Anal. **9** (2003), no. 2, 275–286.

[7] M. Balcerzak, A. Bartoszewicz, and K. Ciesielski, *On Marczewski-Burstin representations of certain algebras of sets*, Real Anal. Exchange **26** (2000/01), no. 2, 581–591.

[8] M. Balcerzak, A. Bartoszewicz, and P. Koszmider, *On Marczewski-Burstin representable algebras*, Colloq. Math. **99** (2004), no. 1, 55–60.

[9] M. Balcerzak, K. Dems, and A. Komisarski, *Statistical convergence and ideal convergence for sequences of functions*, J. Math. Anal. Appl. **328** (2007), no. 1, 715–729.

[10] A. Bartoszewicz, *Marczewski-Burstin representations of Boolean algebras isomorphic to a power set*, Bull. Pol. Acad. Sci. Math. **53** (2005), no. 3, 239–250.

[11] A. Bartoszewicz and K. Ciesielski, *MB-representations and topological algebras*, Real Anal. Exchange **27** (2001/02), no. 2, 749–755.

[12] A. Bartoszewicz and A. Kowalski, *The Stone representation of an atomic complete Boolean algebra is Marczewski-Burstin representable*, J. Appl. Anal. **18** (2012), no. 2, 297–301.

[13] J. E. Baumgartner, *Ultrafilters on ω*, J. Symbolic Logic **60** (1995), no. 2, 624–639.

[14] T. Bermúdez and A. Martinón, *Changes of signs in conditionally convergent series on a small set*, Appl. Math. Lett. **24** (2011), no. 11, 1831–1834.

[15] K. P. S. Bhaskara Rao and M. Bhaskara Rao, *Theory of charges*, Pure and Applied Mathematics, vol. 109, Academic Press, Inc. [Harcourt Brace Jovanovich, Publishers], New York, 1983. A study of finitely additive measures, With a foreword by D. M. Stone.

[16] A. Blass, *Combinatorial cardinal characteristics of the continuum*, Handbook of set theory. Vols. 1, 2, 3, 2010, pp. 395–489.

[17] P. Borodulin-Nadzieja, B. Farkas, and G. Plebanek, *Representations of ideals in Polish groups and in Banach spaces*, arXiv:1402.0441v1 (2014). (to appear in J. Symbolic Logic).

[18] D. Borzestowski and I. Recław, *On Lunina's 7-tuples for ideal convergence*, Real Anal. Exchange **35** (2010), no. 2, 479–485.

[19] J. Brendle and D. A. Mejía, *Rothberger gaps in fragmented ideals*, Fund. Math. **227** (2014), no. 1, 35–68.

[20] J. B. Brown and H. Elalaoui-Talibi, *Marczewski-Burstin-like characterizations of σ-algebras, ideals, and measurable functions*, Colloq. Math. **82** (1999), no. 2, 277–286.

[21] L. Bukovský, P. Das, and J. Šupina, *Ideal quasi-normal convergence and related notions*, 2014. manuscript.

[22] L. Bukovský, I. Recław, and M. Repický, *Spaces not distinguishing pointwise and quasinormal convergence of real functions*, Topology Appl. **41** (1991), no. 1-2, 25–40.

[23] L. Bukovský, I. Recław, and M. Repický, *Spaces not distinguishing convergences of real-valued functions*, Topology Appl. **112** (2001), no. 1, 13–40.

[24] H. Cartan, *Filtres et ultrafiltres.*, C. R. Acad. Sci., Paris **205** (1937), 777–779.

[25] K. Ciesielski and J. Pawlikowski, *Nice Hamel bases under the covering property axiom*, Acta Math. Hungar. **105** (2004), no. 3, 197–213.

[26] K. Ciesielski and J. Pawlikowski, *The covering property axiom, CPA*, Cambridge Tracts in Mathematics, vol. 164, Cambridge University Press, Cambridge, 2004. A combinatorial core of the iterated perfect set model.

[27] Á. Császár and M. Laczkovich, *Discrete and equal convergence*, Studia Sci. Math. Hungar. **10** (1975), no. 3-4, 463–472 (1978).
[28] Á. Császár and M. Laczkovich, *Discrete and equal Baire classes*, Acta Math. Hungar. **55** (1990), no. 1-2, 165–178.
[29] P. Das, S. Dutta, and S. K. Pal, *On $\mathcal{I}$ and $\mathcal{I}^*$-equal convergence and an Egoroff-type theorem*, Mat. Vesnik **66** (2014), no. 2, 165–177.
[30] N. G. de Bruijn, *Functions whose differences belong to a given class*, Nieuw Arch. Wiskunde (2) **23** (1951), 194–218.
[31] G. Debs and J. Saint Raymond, *Filter descriptive classes of Borel functions*, Fund. Math. **204** (2009), no. 3, 189–213.
[32] A. Dow, *The space of minimal prime ideals of $C(\beta\mathbf{N} - \mathbf{N})$ is probably not basically disconnected*, General topology and applications (Middletown, CT, 1988), 1990, pp. 81–86.
[33] L. Drewnowski and T. Łuczak, *On nonatomic submeasures on* $\mathbb{N}$, Arch. Math. (Basel) **91** (2008), no. 1, 76–85.
[34] L. Drewnowski and T. Łuczak, *On nonatomic submeasures on* $\mathbb{N}$. *II*, J. Math. Anal. Appl. **347** (2008), no. 2, 442–449.
[35] L. Drewnowski and T. Łuczak, *On nonatomic submeasures on* $\mathbb{N}$. *III*, Arch. Math. (Basel) **92** (2009), no. 4, 377–382.
[36] L. Drewnowski and P. J. Paúl, *The Nikodým property for ideals of sets defined by matrix summability methods*, Rev. R. Acad. Cienc. Exactas Fís. Nat. (Esp.) **94** (2000), no. 4, 485–503. Perspectives in mathematical analysis (Spanish).
[37] R. Estrada and R. P. Kanwal, *Series that converge on sets of null density*, Proc. Amer. Math. Soc. **97** (1986), no. 4, 682–686.
[38] I. Farah, *Analytic quotients: theory of liftings for quotients over analytic ideals on the integers*, Mem. Amer. Math. Soc. **148** (2000), no. 702, xvi+177.
[39] I. Farah, *How many Boolean algebras $\mathcal{P}(\mathbb{N})/\mathcal{I}$ are there?*, Illinois J. Math. **46** (2002), no. 4, 999–1033.
[40] B. Farkas and L. Soukup, *More on cardinal invariants of analytic P-ideals*, Comment. Math. Univ. Carolin. **50** (2009), no. 2, 281–295.
[41] J. Flašková, *Ideals and sequentially compact spaces*, Topology Proc. **33** (2009), 107–121.
[42] J. Flašková, *Ultrafilters and small sets*, Ph.D. Thesis (Charles University, Prague), 2006.
[43] J. Flašková, *Ultrafilters on ω*, Proc. RIMS **1619** (2006).
[44] P. Frankl, R. L. Graham, and V. Rödl, *Iterated combinatorial density theorems*, J. Combin. Theory Ser. A **54** (1990), no. 1, 95–111.
[45] H. Fujita, *Remarks on two problems by M. Laczkovich on functions with Borel measurable differences*, Acta Math. Hungar. **117** (2007), no. 1-2, 153–160.
[46] H. Fujita and T. Mátrai, *On the difference property of Borel measurable functions*, Fund. Math. **208** (2010), no. 1, 57–73.
[47] H. Furstenberg and B. Weiss, *Topological dynamics and combinatorial number theory*, J. Analyse Math. **34** (1978), 61–85 (1979).
[48] O. Guzmán-González and D. Meza-Alcántara, *Some structural aspects of the Katětov order on Borel ideals*, to appear in Order (2015), DOI: 10.1007/s11083-015-9358-8.
[49] E. Helly, *Über lineare Funktionaloperationen*, Sitzungsber. der Kaiserl. Akad. der Wiss. in Wien, Math.-naturwiss. Kl. **121** (1912), 265–297.
[50] F. Hernández-Hernández and M. Hrušák, *Cardinal invariants of analytic P-ideals*, Canad. J. Math. **59** (2007), no. 3, 575–595.
[51] N. Hindman, *Finite sums from sequences within cells of a partition of n*, Journal of Combinatorial Theory, Series A **17** (1974), no. 1, 1–11.
[52] M. Hrušák, *Combinatorics of filters and ideals*, Set theory and its applications, 2011, pp. 29–69.
[53] A. L. Jones, *A brief remark on van der Waerden spaces*, Proc. Amer. Math. Soc. **132** (2004), no. 8, 2457–2460.

[54] V. Kadets and A. Leonov, *Dominated convergence and Egorov theorems for filter convergence*, Zh. Mat. Fiz. Anal. Geom. **3** (2007), no. 2, 196–212, 284.
[55] M. Katětov, *On descriptive classification of functions*, General topology and its relations to modern analysis and algebra, III (Proc. Third Prague Topological Sympos., 1971), 1972, pp. 235–242.
[56] M. Katětov, *Products of filters*, Comment. Math. Univ. Carolinae **9** (1968), 173–189.
[57] M. Katětov, *On descriptive classes of functions*, Theory of sets and topology (in honour of Felix Hausdorff, 1868–1942), 1972, pp. 265–278.
[58] P. Klinga, *Rearranging series of vectors on a small set*, J. Math. Anal. Appl. **424** (2015), no. 2, 966–974.
[59] M. Kojman, *Hindman spaces*, Proc. Amer. Math. Soc. **130** (2002), no. 6, 1597–1602.
[60] M. Kojman, *van der Waerden spaces*, Proc. Amer. Math. Soc. **130** (2002), no. 3, 631–635.
[61] M. Kojman and S. Shelah, *van der Waerden spaces and Hindman spaces are not the same*, Proc. Amer. Math. Soc. **131** (2003), no. 5, 1619–1622.
[62] P. Kostyrko, T. Šalát, and W. Wilczyński, *$\mathcal{I}$-convergence*, Real Anal. Exchange **26** (2000/01), no. 2, 669–685.
[63] M. Kuczma, *An introduction to the theory of functional equations and inequalities*, Second, Birkhäuser Verlag, Basel, 2009. Cauchy's equation and Jensen's inequality, Edited and with a preface by Attila Gilányi.
[64] A. Kwela, *Kombinatoryczne i deskryptywne własności ideałów na zbiorach przeliczalnych*, Ph.D. Thesis (Instytut Matematyczny Polskiej Akademii Nauk), 2014.
[65] A. Kwela, *A note on a new ideal*, J. Math. Anal. Appl. **430** (2015), no. 2, 932–949.
[66] M. Kysiak, *Nonmeasurable algebraic sums of sets of reals*, Colloq. Math. **102** (2005), no. 1, 113–122.
[67] M. Laczkovich, *Functions with measurable differences*, Acta Math. Acad. Sci. Hungar. **35** (1980), no. 1-2, 217–235.
[68] M. Laczkovich, *The difference property*, Paul Erdős and his mathematics, I (Budapest, 1999), 2002, pp. 363–410.
[69] M. Laczkovich, *Two constructions of Sierpiński and some cardinal invariants of ideals*, Real Anal. Exchange **24** (1998/99), no. 2, 663–676.
[70] M. Laczkovich and I. Recław, *Ideal limits of sequences of continuous functions*, Fund. Math. **203** (2009), no. 1, 39–46.
[71] C. Laflamme, *Filter games and combinatorial properties of strategies*, Set theory (Boise, ID, 1992–1994), 1996, pp. 51–67.
[72] A. Louveau, *Sur la génération des fonctions boréliennes fortement affines sur un convexe compact métrisable*, Ann. Inst. Fourier (Grenoble) **36** (1986), no. 2, 57–68.
[73] M. A. Lunina, *Sets of convergence and divergence of sequences of real-valued continuous functions on a metric space*, Mat. Zametki **17** (1975), 205–217.
[74] A. R. D. Mathias, *Solution of problems of Choquet and Puritz*, Conference in Mathematical Logic—London '70 (Bedford Coll., London, 1970), 1972, pp. 204–210. Lecture Notes in Math., Vol. 255.
[75] T. Mátrai, *Weak difference property of functions with the Baire property*, Fund. Math. **177** (2003), no. 1, 1–17.
[76] G. Matusik and T. Natkaniec, *Algebraic properties of Hamel functions*, Acta Math. Hungar. **126** (2010), no. 3, 209–229.
[77] G. Matusik, *On the lattice generated by Hamel functions*, Real Anal. Exchange **36** (2010/11), no. 1, 65–77.
[78] K. Mazur, *F_σ-ideals and $\omega_1\omega_1^*$-gaps in the Boolean algebras $P(\omega)/I$*, Fund. Math. **138** (1991), no. 2, 103–111.
[79] S. Mazurkiewicz, *Sur les suites de fonctions continues*, Fund. Math. **18** (1932), 114–117.
[80] D. Meza-Alcántara, *Ideals and filters on countable set*, Ph.D. Thesis (Universidad Nacional Autónoma de México), 2009.
[81] S. Mrówka, *On completely regular spaces*, Fund. Math. **41** (1954), 105–106.

[82] N. Mrożek, *Ideal version of Egorov's theorem for analytic P-ideals*, J. Math. Anal. Appl. **349** (2009), no. 2, 452–458.
[83] N. Mrożek, *Zbieżność idealowa ciągów funkcyjnych*, Ph.D. Thesis (Uniwersytet Gdański), 2010.
[84] J. Mycielski, *Independent sets in topological algebras*, Fund. Math. **55** (1964), 139–147.
[85] T. Natkaniec, *An example of a quasi-continuous Hamel function*, Real Anal. Exchange **36** (2010/11), no. 1, 231–236.
[86] T. Natkaniec, *On Hamel-like functions*, Real functions, density topology and related topics, 2011, pp. 122–129.
[87] T. Natkaniec and P. Szuca, *On the ideal convergence of sequences of quasi-continuous functions*, to appear in Fund. Math. (2015).
[88] T. Natkaniec and J. Wesołowska, *Sets of ideal convergence of sequences of quasi-continuous functions*, J. Math. Anal. Appl. **423** (2015), no. 2, 924–939.
[89] M. Paštéka, *Convergence of series and submeasures of the set of positive integers*, Math. Slovaca **40** (1990), no. 3, 273–278.
[90] K. Płotka, *On functions whose graph is a Hamel basis*, Proc. Amer. Math. Soc. **131** (2003), no. 4, 1031–1041.
[91] K. Płotka, *Darboux-like functions within the class of Hamel functions*, Real Anal. Exchange **34** (2009), no. 1, 115–125.
[92] K. Płotka, *On functions whose graph is a Hamel basis. II*, Canad. Math. Bull. **52** (2009), no. 2, 295–302.
[93] K. Płotka and I. Recław, *Finitely continuous Hamel functions*, Real Anal. Exchange **30** (2004/05), no. 2, 867–870.
[94] I. Recław, *Metric spaces not distinguishing pointwise and quasinormal convergence of real functions*, Bull. Polish Acad. Sci. Math. **45** (1997), no. 3, 287–289.
[95] I. Recław, *Sets of filter convergence of sequences of continuous functions*, J. Math. Anal. Appl. **394** (2012), no. 2, 475–480.
[96] I. J. Schoenberg, *The integrability of certain functions and related summability methods.*, Amer. Math. Monthly **66** (1959), 361–375.
[97] J. J. Sember and A. R. Freedman, *On summing sequences of 0's and 1's*, Rocky Mountain J. Math. **11** (1981), no. 3, 419–425.
[98] W. Sierpiński, *Sur la Question de la Mesurabilité de la Base de M. Hamel*, Fund. Math. **1** (1920), no. 1, 105–111.
[99] W. Sierpiński, *Sur les suites transfinies convergentes de fonctions de Baire*, Fund. Math. **1** (1920), no. 1, 134–141.
[100] S. Solecki, *Analytic ideals and their applications*, Ann. Pure Appl. Logic **99** (1999), no. 1-3, 51–72.
[101] S. Solecki, *Filters and sequences*, Fund. Math. **163** (2000), no. 3, 215–228.
[102] H. Steinhaus, *Comptes rendus: Société Polonaise de Mathématique. Section de Wroclaw. Septembre 1948-Mars 1949*, Colloquium Math. **2** (1951), 63–78.
[103] J. Steprāns, *Reaping numbers in quotient algebras*, 2013. manuscript.
[104] M. Talagrand, *Compacts de fonctions mesurables et filtres non mesurables*, Studia Math. **67** (1980), no. 1, 13–43.
[105] B. L. van der Waerden, *Beweis eine baudetschen vermutung*, Nieus Arch. Wisk. **15** (1927), 212–216.
[106] E. K. van Douwen, *The integers and topology*, Handbook of set-theoretic topology, 1984, pp. 111–167.
[107] W. Wilczyński, *On Riemann derangement theorem*, Słupskie Prace Matematyczno-Fizyczne **4** (2007), 79–82.

Gdańsk, 18 sierpnia 2015

Rafał Filipów

Rafał Filipów