Report on the thesis

Quantitative approach to some aspects of Bell nonlocality and contextuality

by

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The thesis of **Pankajkumar Joshi** submitted in October 2015 to the Faculty of Mathematics, Physics and Informatics of University of Gdańsk for the Ph.D. degree consists of 116 pages of text divided into five chapters and three appendices. The thesis, written in English and based on four papers published in leading international journals (including *Physical Review Letters* and *Physical Review* A), deals with foundations of the quantum theory. In particular, major part of the thesis concern Bell nonlocality for non-signalling theories and quantum contextuality.

The first chapter of the thesis contains a short introduction to the formalism of quantum theory and presents necessary concepts and definitions. It presents also the structure and the aim of the thesis and contains a brief description of the main results achieved by the author.

A family of Bell inequalities for a class of probability distributions, called *boxes*, is analysed in chapter 2. The author derives lower and upper bounds for the new quantity, called *fraction of determinism*, which characterizes the relative weight of the deterministic component in a convex mixture of probability distributions analysed. This notion is useful to analyse quantitatively the Bell inequalities and to bound the difference between classical and quantum values. Furthermore, the fraction of determinism is helpful to study the classical – quantum correspondence, as this quantity is by construction positive for any classical probability distribution, while it can be zero for a quantum box. It is worth to stress here that

an analogous lower bound obtained in the thesis for the *classical fraction*, describes the maximal weight of a local box in the mixture of box probability distributions.

A rather general property concerning mixed quantum states, called *reverse triangular inequality*, is formulated and established in this chapter. This result – a lower bound for the distance between a given mixed state and the barycentre of an ensemble of other states – can be considered as a one of key results of the thesis. Interestingly, under some technical assumptions the bound becomes stronger if the density matrices in question have classical properties and do commute. It would be good to know, whether analogous inequalities can also be established for other distances including e.g. the trace distance or the Bures distance.

Reverse triangular inequality was used to establish a concrete bound for the fraction of determinism in the case of an arbitrary probability distribution corresponding to the 2 x N inputs. It could be interesting to analyse, whether the corresponding inequalities [say (2.41) and (2.70)] can be further improved. Moreover, one can raise the question, how the results could be generalized for the case of a larger number of inputs for the first party (e.g. for the case of M x N problem).

Chapter 3 of the thesis is devoted to no-broadcasting of nonlocal boxes. Mr Joshi shows that it is not possible to make a 2-copy broadcast of a 2 x 2 nonlocal box. This result implies in turn impossibility of N-copy broadcasting.

A quantitative approach to analyse quantum contextuality is advocated in chapter 4. To investigate the strength of contextuality the author uses mutual information of contextuality. A special attention is paid to the uniform relative entropy of contextuality which is shown to be 2-copy additive for the considered family of isotropic boxes. These results provide a novel significant contribution to the theory of contextuality – a less studied resource, which can also be used for quantum information processing. To this end the author proves Theorem 4.3 which establishes existence of a noncontextuality preserving map useful

to destill contextuality, constructs an explicit protocol for distilling contextuality and derives an upper bound on the distillation rate.

All results of the thesis are summarized in the last chapter 5. However, it consists of two pages only, and does not contain any broader list of open problems nor it does not present possibilities for further studies. For instance, any remarks on the class of probability distributions, for which various measures of contextuality admit maximal values, and any relations to the degree of their nonlocality or entanglement would be welcome. The thesis contains also valuable appendices, which for completeness contain a full proof of the reverse triangle inequality and a short review of the measures of contextuality, taken from scientific papers co-authored by Mr Joshi. The list of references provides a representative selection of items from a broad literature of the subject. All chapters of the thesis begin with interesting quotations from masters of the quantum theory related to the work.

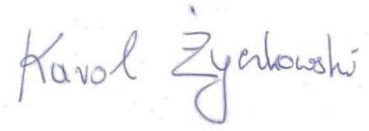
The thesis concerning foundations of the quantum theory belongs to mathematical physics and it is written in a mathematically rigorous way. The author uses various algebraic techniques and presents a good knowledge of several branches mathematics and shows excellent analytic skills, which allow him to prove some useful theorems. It is worth to emphasize that the paper on the project on reverse triangle inequality, was written jointly with mathematicians, so the author has an experience in such a collaboration.

The entire thesis is written clearly and in general it is easy to understand and follow. However, some remarks can be made concerning the quality of the editorial work done. Although some Figures in the thesis (e.g. Fig. 4.4) are well prepared and informative, some others are suboptimal. For instance Fig 3.1 and 3.2 are decorated with hardly legible (too small) labels at figure axes. Fig 4.2 larger and easier to follow, but the caption will not be easy to understand by a less experienced reader. (E.g.: what are the relations between Adversary A, Alice, Sender S, Bob, Box B and Receiver R?), while Fig. 4.3 contains references to Observations 3.1 and 3.2 instead of Observations 4.1 and 4.2. It looks like the

spelling checker was used in the thesis only sporadically (see ‘thoery’ in the Abstract, p. iv). Furthermore, the list of references was compiled in haste and rather carelessly - the names of Bell, Einstein, Podolsky, Rosen, Kochen and Specker are often written with miniscules – see ref. [ASDR82, AGR81, AGR82, Ara04b, Bel64HC14, CAE+14, JHH+15, Mer93, Tsi80] and some other references. There are misprints in the name of an author ([BDSW96, BGP+96]), the name of the journal ([HHHH09, JHH+15]) or the title of the paper ([LS10,vDGG05)], not to mention not entirely correct spelling of the German title in [Spe60]. It seems likely that the usage of the norms $\|.\|$ and $\|.\|_1$ is not entirely consistent throughout the entire thesis including the appendices.

The minor critical comments raised above concern only editorial issues and certainly do not diminish the overall high value of the entire Ph. D. dissertation. Several firm results on Bell non-locality and quantitative characterization of quantum contextuality deserve to be appreciated. To the best of my knowledge the thesis of Pankajkumar Joshi is original, sound and correct.

Let me conclude this report stating explicitly that I do appreciate the scientific merit of the results presented, which may be regarded as a valuable contribution to the theory of quantum information. Thus I believe, the thesis refereed meets all international standards and I am pleased to recommend to proceed with further steps of the Ph.D. procedure of Mr. **Pankajkumar Joshi**.



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