Abstract

By an *ideal on* ω we mean a nonempty family of subsets of ω closed under taking finite unions and subsets of its elements. Moreover, we also assume that ideals contain all finite sets. By Fin we denote the ideal of all finite subsets of ω.

Let $\mathcal{I}$ be an ideal on ω. We say that a sequence $(x_n)_{n\in\omega}$ of reals is $\mathcal{I}$*-convergent* to $x \in \mathbb{R}$ if the set $\{n \in \omega : |x_n - x| \geqslant \varepsilon\} \in \mathcal{I}$ for every $\varepsilon > 0$ (see [6]).

In the Ph.D. thesis we consider the following definition of ideal equal convergence of a sequence of functions (we use the name e-convergence instead of equal convergence for simplicity). Let $\mathcal{I}, \mathcal{J}$ be ideals on ω. Assume that f_n $(n \in \omega)$ and f are real-valued functions defined on a set X. We say that the sequence $(f_n)_{n\in\omega}$ is $(\mathcal{I}, \mathcal{J})$*-equally convergent* to f if there exists a sequence of positive reals $(\varepsilon_n)_{n\in\omega}$ such that $(\varepsilon_n)_{n\in\omega}$ is $\mathcal{J}$-convergent to 0 and $\{n \in \omega : |f_n(x) - f(x)| \geqslant \varepsilon_n\} \in \mathcal{I}$ for every $x \in X$ (see [4]).

The above definition generalizes equal convergence introduced by Császár, Laczkovich (see [1]) and two different kinds of ideal equal convergence introduced by Das, Dutta, Pal (see [2]) and Filipów, Szuca (see [5]).

We prove a characterization showing when the ideal equal limit is unique. We study relationships between ideal equal convergence and various kinds of other ideal convergences of sequences of real functions. We also solve a few problems posed in [2] and show how the results obtained by Šupina in [12] can be easily concluded from our results.

Ideal pointwise convergence (see [6]) is a generalization of well known statistical convergence introduced by Schoenberg (see [10]) and Steinhaus (see [11]). Among other we prove a characterization showing when the ideal pointwise convergence implies the ideal equal convergence. The mentioned characterization is expressed in terms of cardinal coefficients related to the bounding number $\mathfrak{b}$. We compute the value of those coefficients for some classes of ideals, for instance for ideals with hereditary Baire property.

Laczkovich and Recław (see [7]) and (independently) Debs and Saint Raymond (see [3]) characterized first Baire class with respect to ideal convergence (the family of pointwise ideal limits of sequences of continuous functions) for every Borel ideal and Polish space. In particular, they characterized Borel ideals for which the first Baire class with respect to ideal convergence is equal to the classical first Baire class. Filipów and Szuca (see [5]) have extended this result to ideal discrete convergence and $(\mathcal{I}, \text{Fin})$-equal convergence. Moreover, they characterized the ideals for

which higher Baire classes in the case of all three considered notions of convergence (ideal, ideal discrete and $(\mathcal{I}, \mathrm{Fin})$-equal convergence) coincide with the classical Baire classes for all perfectly normal topological spaces. We generalize their results to $(\mathcal{I}, \mathcal{J})$-equal convergence. We characterize Baire classes in the case of $(\mathcal{I}, \mathcal{J})$-equal convergence for every pair of ideals $(\mathcal{I}, \mathcal{J})$, where $\mathcal{I}$ is coanalytic.

Recently, Natkaniec and Szuca (see [8] and [9]) obtained similar results in the case of quasi-continuous functions instead of continuous functions. Namely, they characterized Baire systems generated by the family of quasi-continuous functions in the case of ideal convergence and ideal discrete convergence for all Borel ideals and metric Baire spaces. We characterize Baire systems generated by quasi-continuous functions in the case of $(\mathcal{I}, \mathcal{J})$-equal convergence for every pair of ideals $(\mathcal{I}, \mathcal{J})$, where $\mathcal{I}$ is Borel.

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