Abstract of the PhD thesis

Estimation on the sphere

Natalia Jarzębkowska

In this dissertation, we focus on the adaptive estimation of a density function on the $d-$dimensional unit sphere $\mathbb{S}^d$, $d \geq 2$ using the new type of spherical frames.

We start by collecting information about the classical Sobolev and Besov spaces on the sphere. Then we unify and simplify the approach to estimation of the density in L^2 arising from Talagrand's inequality. Finally, we give the construction (which is alternative to the widely used needlets) of Parseval frame based on the methods introduced in Bownik M., Dziedziul K. (2015). Smooth Orthogonal Projections on Sphere. *Constr. Approx.*, 41(1), 23-48. In doing so, we employ two smooth orthogonal projections with good localization properties and appropriate stereographic operators. The use of these operators aims to present the Bownik-Dziedziul frame construction in a more accessible way, and thereby better understandable for a wider group of specialists.

Eventually, using the frame, we give the characterization of Besov space $B^s_{2,\infty}(\mathbb{S}^d)$ by the frame coefficients, construct frame estimator and show that it achieves the optimal rate of convergence on a function class defined in terms of the Besov space without knowing the parameters of the model, i.e. it is so-called adaptive estimator.

Keywords: sphere, frames, density estimation, adaptive estimation, Besov space, Talagrand's inequality