Abstract in English

author: Paweł Barbarski

title: Various generalizations and applications of the Sharkovsky Theorem on coexistence of periodic orbits of continuous mappings

The Sharkovsky Theorem states what are the possible sets of periods of periodic points of a continuous real function. Thus, it belongs to the area of discrete dynamical systems (combinatorial dynamics), but also to the real functions theory. Nevertheless, multiple directions of generalizations and applications of the theorem go beyond those areas. Here we concentrate on two directions of those, thus the results of the dissertation can be divided into two categories.

Firstly, we formulate some new randomized Sharkovsky-type results and show their applications. We can extend a discrete dynamical system with an additional dimension, and assume some measurability conditions on it, while preserving some continuity conditions on the original dimension. In such a way we arrive at a notion of a random operator and its random orbit. In the dissertation we generalize some known randomization results. We achieve that by significantly generalizing the characterization results on the random operators (the so called "transformation to the deterministic case" method), through using stronger measurability arguments, including selection theorems. Furthermore, we formulate some new kind of Sharkovsky Theorem analogues and use the results in the area of random differential inclusions.

Secondly, we analyze algebraic properties of functions fulfilling the assertion of the Sharkovsky Theorem. We solve some published problem in that area, actually generalizing it significantly. The solution of the problem is the characterization of all continuous functions belonging to an Aumann ring generated by any given Darboux function. We arrive at the final solution through multiple steps providing solutions to some simpler cases of the problem, and through some real functions theory reasoning.