Abstract

In the Ph.D. thesis we consider several different classes of ideals and investigate their properties. We are mostly interested in properties related to ideal convergence and structure of ideals. The second main point of the thesis is to study density functions that can be used to define ideals.

We investigate ideal convergence when considering BW-like properties (introduced by Filipów, Mrożek, Recław and Szuca in [5]) for several ideals connected with well known combinatorial theorems of Hindman and van der Waerden. We expand the results of Kojman (see [11], [12]) by showing that several of these properties coincide for examined ideals and treat this result as a starting point for our studies of the homogeneity of ideals. We present several examples of homogeneous ideals (term analogous to homogeneous filters introduced by Fremlin in [6]) and show connections between homogeneous ideals and K-uniform ideals, which answers several questions posed by Hrušák in [9] and Meza-Alcántara in [14]. Moreover, we apply homogeneity to solve several problems about $\mathcal{I}$-invariant functions stated by Balcerzak, Głąb and Swaczyna in [2].

The best known connections between ideals and densities of subsets of natural numbers are classical ideals of asymptotic density zero and uniform density zero. There are also whole classes of ideals defined by density functions like Farah density ideals from [4], Erdős-Ulam defined in [10] by Just and Krawczyk and matrix summability ideals introduced by Drewnowski and Paúl in [3]. In the thesis we consider both some of these mentioned classes and new classes of ideals defined by other kinds of densities like weighted uniform density introduced by Giuliano Antonini and Grekos in [8] and simple density presented in [1] by Balcerzak, Das, Filipczak and Swaczyna. We show several properties of these new classes of ideals, examine their topological structure and compare them with well known ideals.

Investigation into ideals defined by densities naturally leads to our study on densities of subsets of natural numbers themselves. We compare different weighted uniform densities with classic uniform density and characterize when they have the Darboux property, which solves the problem of Mačaj, Sleziak and Toma from [13]. Furthermore, we tackle numerous problems on densities stated by Giuliano, Grekos and Mišík in [7]. For example, we characterize the uniform distribution of ratio block sequences that were first considered by Strauch and Tóth in [15] as a continuation of Šalát's studies of ratio sets (see [16]).

Bibliografia

[1] Marek Balcerzak, Pratulananda Das, Małgorzata Filipczak, Jarosław Swaczyna, *Generalized kinds of density and the associated ideals*, Acta Math. Hungar. **147** (2015), no. 1, 97–115.

[2] Marek Balcerzak, Szymon Głąb, Jarosław Swaczyna, *Ideal invariant injections*, J. Math. Anal. Appl. **445** (2017), no. 1, 423–442.

[3] Lech Drewnowski, Pedro J. Paúl, *The Nikodým property for ideals of sets defined by matrix*, Rev. R. Acad. Cienc. Exactas Fís. Nat. (Esp.) **94** (2000), no. 4, 485–503, Perspectives in mathematical analysis (Spanish).

[4] Ilijas Farah, *Analytic quotients: theory of liftings for quotients over analytic ideals on the integers*, Mem. Amer. Math. Soc. **148** (2000), no. 702, xvi+177.

[5] Rafał Filipów, Nikodem Mrożek, Ireneusz Recław, and Piotr Szuca, *Ideal convergence of bounded sequences*, J. Symbolic Logic **72** (2007), no. 2, 501–512.

[6] David Fremlin, *Filters of countable type*, (2007), dostępne pod adresem http://www.essex.ac.uk/maths/people/fremlin/preprints.htm, ostatnia wizyta na stronie w listopadzie 2017.

[7] Rita Giuliano, Georges Grekos, Ladislav Mišík, *Open problems on densities II*, Diophantine analysis and related fields 2010, AIP Conf. Proc., vol. 1264, Amer. Inst. Phys., Melville, NY, 2010, pp. 114–128.

[8] Rita Giuliano Antonini, Georges Grekos, *Weighted uniform densities*, J. Théor. Nombres Bordeaux **19** (2007), no. 1, 191–204.

[9] Michael Hrušák, *Katětov order on Borel ideals*, Archive for Math. Logic **56** (2017), no. 7, 831–847.

[10] Winfried Just, Adam Krawczyk, *On certain Boolean algebras $\mathcal{P}(\omega)/I$*, Trans. Amer. Math. Soc. **285** (1984), no. 1, 411–429.

[11] Menachem Kojman, *Hindman spaces*, Proc. Amer. Math. Soc. **130** (2002), no. 6, 1597–1602 (electronic).

[12] ———, *van der Waerden spaces*, Proc. Amer. Math. Soc. **130** (2002), no. 3, 631–635 (electronic).

[13] Martin Mačaj, Martin Sleziak, Vladimír Toma, *On weighted uniform density*, Unif. Distrib. Theory **3** (2008), no. 2, 101–127.

[14] David Meza-Alcántara, *Ideals and filters on countable sets*, Ph.D. thesis, Universidad Nacional Autónoma de México, 2009.

[15] Oto Strauch, János T. Tóth, *Distribution functions of ratio sequences*, Publ. Math. Debrecen **58** (2001), no. 4, 751–778.

[16] Tibor Šalát, *On ratio sets of sets of natural numbers*, Acta Arith. **15** (1968/1969), 273–278.