Abstract

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Multi-state two-dimensional number-conserving cellular automata

Cellular Automata are systems often used to model physical phenomena. While users relying on simulation often use as many states as convenient, theorists concentrate themselves on the simplest binary case. This work is a step in the direction of theoretical understanding of cellular automata with more than two states. Concentrating on the von Neumann neighborhood and some other common and natural restrictions, we are able to give complete lists of rules in several cases. Such lists sometimes lead to new discoveries of ways of designing models as well as a better understanding of limitations.

This work presents an approach for the generation of all number-conserving two-dimensional cellular automata (CAs) with the von Neumann neighborhood in several special cases. Together with basic definitions, Chapter 2 shows possible intersections of von Neumann neighborhoods of two cells. Throughout the thesis, we take advantage of the fact that two distinct neighborhoods have at most two cells in common. In Chapter 3 we present the necessary and sufficient conditions for a local rule to be number-conserving. These conditions are then exploited in more specific situations. We are able to completely describe all totalistic and outer-totalistic NCCAs. Furthermore, we consider a wider class of symmetric and outer-symmetric CAs and give a detailed description. In Chapter 4 we give a description of all three-state two-dimensional rules and their equivalence classes. In Chapter 5 we describe an important tool introduced by Wolnik et al. [88], *i.e.*, the split-and-perturb decomposition theorem. We show a way of using this theorem to enumerate all four-state number-conserving CAs. Further increasing the number of states without additional restrictions, *e.g.*, enumerating all five-state rules does not seem possible.

In Chapters 6 and 7 we introduce restrictions in addition to number conservation. These are invertibility (in Chapter 6) and rotational symmetry (in Chapter 7). We briefly return to the one-dimensional setting to present a novel representation of 1D invertible and number-conserving cellular automata with four states. Carrying this view over to two dimensions, we are able to construct 65 four-state invertible and number-conserving 2D cellular automata with the von Neumann neighborhood. A clever use of the split-and-perturb

decomposition theorem allows to prove by elimination that this list is complete. In Chapter 7 we describe CAs with a local rule that is invariant under rotation of the neighborhood by 90 degrees. This allows for an elegant formulation of a necessary condition for number-conservation. Then a series of technical lemmata are formulated and proved. It allows to enumerate and give a parametric description of all CAs in the considered class with up to seven states. We complete the thesis with a short chapter with conclusions and future perspectives on the considered topic.